Game Skill Measure for Mixed Games

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Abstract—Games can be classified as games of skill, games of chance or otherwise be classified as mixed. This paper deals with the topic of scientifically classifying mixed games as more reliant on elements of chance or elements of skill and ways to scientifically measure the amount of skill involved. This is predominantly useful for classification of games as legal or illegal in deferent jurisdictions based on the local gaming laws. We propose a novel measure of skill to chance ratio called the Game Skill Measure (GSM) and utilize it to calculate the skill component of a popular variant of Poker.

Keywords—Chance, Game, Skill, Luck.

I. INTRODUCTION

FROM the moment a human being is born and till the day he dies games are an important part of our everyday life. We first get introduced to games as children trying to learn how to properly act in society. Games help us to learn how to deal with our pears in a competitive environment, how to act with opposite gender in a family unit, how to deal with tools in the workplace and overall help to shape and develop our minds. In fact games are often completely assigned to the domain of children since they are the once who always up for a good game and can benefit the most from it.

However, games are not only for children. Throughout history adult humans have used games to entertain themselves and others. Games are used as military exercises and professional training in many disciplines. Scientists often study complex phenomena by simplifying it and representing as a game with well-defined rules making it possible to control for different conditions and draw conclusions and find solutions relatable to the real word problem on which the game is based. Poker is a perfect example of such a game as it is often used to model concepts in communication, economy, psychology and behavioral dynamics.

Numerous way of classifying games exist. Each approach has its own advantages and disadvantages and a lot depends on the ultimate goal of the classifier. One frequently used game taxonomy is based on three broad categories, which are representative of the type of challenge the game provides. They are:

- Games of skill
    - Physical-skill games
    - Mental-skill games
- Games of chance
- Mixed games [7]

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Many games combine elements of skill and strategy with some elements of chance making their classification somewhat complicated. This presents some interesting problems particularly from the point view of the law. Organized gambling or profiting from setting up facilities for playing games of chance for money is illegal in most areas, but games of skill can be legally played for money almost everywhere. This brings up an interesting problem of classifying particular games as games of chance or games of skill. Some cases are clear-cut such as chess, which is no doubt a game of skill versus Craps, which is a game of pure chance. But certain games such as poker have been a subject of debate for many years. With huge amounts of money at stake we would like to see a scientific approach to determining if a game fits a legal definition for a game of skill.

We will start by defining what is meant by the legal concept of Game of Chance. "The term 'game of chance' has an accepted meaning established by numerous adjudications. Although different language is used in some of the cases in defining the term, the definitions are substantially the same. It is the character of the game rather than a particular player's skill or lack of it that determines whether the game is one of chance or skill. The test is not whether the game contains an element of chance or an element of skill but which of them is the dominating factor in determining the result of the game"[4].

For a game to be considered a game of skill, skill needs to predominate over the element of chance. The legal definition for skill is a presence of the following factors, alone or in combination with one another:

1) "A learned power of doing a thing competently;
2) A particular craft, art, ability, strategy, or tactic;
3) A developed or acquired aptitude or ability;
4) A coordinated set of actions, including, but not limited to, eye-hand coordination;
5) Dexterity, fluency, or coordination in the execution of learned physical or mental tasks or both;
6) Technical proficiency or expertise;
7) Development or implementation of strategy or tactics in order to achieve a goal;
8) Knowledge of the means or methods of accomplishing a task" [4].

II. GAME SKILL MEASURE

The question becomes: how can we scientifically measure the element of skill in a game, and express it as a percentage indicating amount of control a player exhibits over the outcome of the game. Once we are able to do so, all that is needed is for the government to pass a law saying any game

with the skill level below a certain threshold is gambling and every other game is a legal game of skill. This would stop all the unnecessary litigation and problems created by the fuzzy nature of game of poker and many others games sharing similar ill-defined character [2]. So for example a game of pure skill would be 100% a game of pure chance would be 0% and all the mixed games would fall in between those two extremes. For simplicity let's call this number *Game Skill Measure* (GSM). Other similar skill measures have been investigated and showed some level of success in classifying mixed type games [1, 3].

To better understand how GSM works and what it really represents lets consider a hypothetical game with GSM of 67%. Suppose the game has only two players. That means that 33% of all instances of the game will be split between the two opponents, each one winning 16.5% of all instances. The remaining 67% of games would be determined by skill. Suppose player A has 1 unit of skill however it was measured and player B is slightly better with 1.2 units of skill. Under our model player A is expected to win 30.45% of instances of the game and player B the remaining 34.55% of the skill determined instances. Giving us a total result of the match between A and B as 46.95/53.05% in the long run. Which is consistent with our expectation of B being a slightly better player.

How the actual units of skill are measured depends on a particular game. For example in chess, go and scrabble rankings are relative so your score only means something in comparison to the scores of other players. And the changes in skill rankings depend on your performance in recent matches. The rankings are structured in such a way as to keep the correlation between difference in player's scores and their chances of winning. This means that:

- Good players don't gain much by beating lesser players
- Lesser players don't lose much by losing to good players
- Good players lose a lot when they lose to lesser players
- Lesser players gain a lot when they beat good players [5]

From the following table we can see how the difference in scores can be used to predict an outcome of the match between two Scrabble players who's rankings are typically from 400-2200 units of skill [5].

TABLE I
PROBABILITY OF A STRONGER PLAYER WINNING BASED ON THE DIFFERENCE IN RATINGS [5]

Difference of Ratings:	Probability of stronger player winning	Probability of weaker player winning
0	50%	50%
50	57%	43%
100	63%	37%
150	70%	30%
200	76%	24%
250	81%	19%
300	85%	15%
400	92%	8%
500	96%	4%

A similar table can be generated for Chess, Go or any other game based on the so-called "Elo" ratings system, named after its inventor Árpád Élő.

It appears from the table above that any player has a chance of winning against any other player regardless of their respective skill level. However games such as Chess, Go and Scrabble are considered to be games of pure skill, since skill clearly dominates over chance if the difference in the skill levels is sufficiently large. This brings up an interesting philosophical question; do games of pure skill exist? Any game widely considered a game of pure skill has some chance elements such as blows of wind in Golf, how tired your opponent is in Chess, and possible physical injuries of your opponent in Bowling. We will not examine this issue any further in this paper, as we are more concerned with practical definitions of games of skill and chance.

In this paper we are often referring to the so-called long run expectation of players, perhaps we should define this concept more precisely. Statistics only apply to significantly large samples. However what is significantly large is not a trivial question. Clearly infinity is a large enough number, but it is not an applicable value to real life situations. We need to find a very large number, which is still possible in terms of human life span. The main conclusion we can make is that a single instance of any mixed game is a pure game of chance, but a million instances may be much heavier dominated by skill. For example if we assume that a full time professional poker player is active for 50 years and plays 40 hours a week at 100 hands per hour (which is high for brick and mortar casinos but possible for online play) he would have engaged in 10,000,000 instances of Poker in his lifetime, which is statistically significant.

III. GSM OF POKER

We have defined skill in general for all types of games; since we are concerned with poker in particular it would make sense to list skills, which are useful in the game of poker, in particular, Texas Hold'em. We will not trivialize the definition of skill by including such basic skills as being able to read card's values and suites, understand basic rules of betting structure and hand rankings. Those abilities are a type of skill but all who play poker as opposed to learn poker already posses those skills in equal proportion. The advanced skills exercised by good poker players are:

1. Ability to calculate precise mathematical odds of a needed card(s) coming on a Turn or River.
2. Ability to understand and predict behavior of opponents often from relatively view observations.

3. Ability to select and exercise a strategy which is dominant to the one believed to be exercised by the opponent.
4. Ability to read opponents behavior and body language to accurately estimate the cards opponent was dealt.
5. Ability to randomize play as to make profiling by other advanced players not possible.
6. Ability to understand and apply advanced strategic concepts such as semi-bluffing and playing for implied odds.

Since as scientists we can only measure and interpret actual behaviors of players not their thought patterns, it is logical for us to equate skill with strategy in the game of poker. It follows that two players who would act exactly the same in a same situation have an equal measure of skill. They would be equally successful or unsuccessful and should be tied in the long run in any game of poker they play against each other.

In a game of pure chance strategy is irrelevant; player has no way of influencing long-term outcome of the game. And all possible outcomes have a predetermined chance of happening depending on random nature of the game. If we can derive a strategy, which allows a player to control the outcome of any poker match, that would constitute a proof for poker being not a pure game of chance. We already know it intuitively but our goal is to approach this problem with a strictly scientific and provable method.

To make our proof complete we will start by precisely describing the game for which the proof is valid. To make the proof as simple as possible while using a real life game we will investigate a game of Texas Hold'em with limited size of bets, particularly its Heads-up variation. Meaning that we only have two players and the bets are structured. The player who is in the position of the dealer is the small blind of 1 unit and the other player is a big blind of 2 units. All other rules are standard Texas Hold'em rules. A match is complete after 100 hands are played between players and the winner is the player with the most units of money at that point. Both players begin with the same amount of money and both players have an equal share of times as a small blind and as a big blind. For simplicity purposes we will refer to this game as Poker100.

In math a proof by existence often relies on a boundary value or an outlier. Since no known strategy for always winning in poker exists, perhaps we can use the other boundary condition, namely the strategy for never wining. By assuming that Poker100 is a game of pure chance, it follows that the percentage of winning and loosing matches should be equal for both players in the long run. Our proof of Theorem 1 is based on proving this to be false.

Theorem 1: Poker100 has GSM > 0%.

Proof: In Poker100 (as in any other Hold'Em game) a player is allowed to fold his hand pre-flop. Suppose the strategy of our player is to try to loose and so he always folds his hand while in a small blind. He is guaranteed to loose half of all hands. Now what happens if our player is in the big blind? It is possible that the other player is also trying to loose (maybe

they both made a bet with a third party?) and so folding from the small blind. Now our player becomes a winner for those hands automatically. In this case both player win five hands each and since no one ever raised, they simply passed the blinds back and forth, they have the same amount of money as then they start, resulting in a draw. The only other alternative is that the second player is not into loosing and plays some other strategy, in that case regardless of his actions be it a raise or a call our player folds his hand resulting in a loss. So as we have demonstrated it is only possible for our player to loose or draw a match but never to win it. He has effectively found a strategy for influencing the outcome of the game. Which proofs that Poker Texas Hold'em, limit, Heads-up variant is not a pure game of chance or in our terminology does not have a GSM of 0%.

Now we will attempt to prove the other boundary case namely that Poker100 is not a pure game of skill.

Theorem 2: Poker100 has GSM < 100%.

Proof: This proof is somewhat more straightforward. Suppose we have two players playing the game of Poker100 once called Lucky and one called Good. Good player has excellent poker skill. Lucky on the other hand never played Poker100 before, but he happens to catch the cards always having the best possible hand at all stages, including Pre-flop, Flop, Turn and River. Since in that situation Lucky will never fold his hand, regardless of actions by Good, it is clear that every hand gets taken to showdown and so is won by Lucky who always has the better hand. This proves that Lucky will always win over Good regardless of strategy or level of skill used by Good. Of course statistics tells us that if we take infinitely many hands of poker two players will have roughly the same number of winners, but in our case we are only dealing with 100 hands of poker so it is mathematically sound to assume that Lucky can get that lucky. We have demonstrated that luck can dominate over skill in the game of Poker100 and so Poker100 is not a game of pure skill, which was to be proven.

Now that we have shown that GSM of Poker100 is: 0 < Poker100 < 1 we need to find a more precise value for the skill measure of Poker100. We will once again try to use an extreme strategy, which may be hard to find in real life games, but yet is perfectly possible. If we analyze all the skills involved in poker and try to imagine the best possible poker player we are essentially faced with someone who knows precisely what two cards his opponent is holding. This is possible as a result of many skills possessed by such a player, such as opponent modeling, reading of body language, and advanced mathematical ability. This is the highest level of skill attainable by a poker player. We do not consider skill, which is not scientifically sounds to be possible, such as predicting flop cards in advance with extra sensory perception.

David Sklansky in his book "The theory of poker" proposes what he calls a fundamental theorem of poker [6]:

"Every time you play a hand differently from the way you would have played it if you could see all your opponents'

cards, they gain; and every time you play your hand the same way you would have played it if you could see all their cards, they lose. Conversely, every time opponents play their hands differently from the way they would have if they could see all your cards, you gain; and every time they play their hands the same way they would have played if they could see all your cards, you lose".

Now since our super player knows his cards and his opponent's cards all he needs to do is put the money in with a better hand and fold with a weaker hand, obviously taking into consideration all the possible outs he has. Both players get an equal number of winning cards in the long run. The times then they have equally good cards do not matter since it results in a split pot and no money exchanges hands. Since our player only bets with a better hand we need to analyze just how much better his hand can be. Anything below 50% will get folded if faced with a raise or a call. A best possible hand with a matching flop represents 100% chance of winning. All other hands fall somewhere in between those two extremes and so in the long run will average somewhere around 75% probability of winning. The situation is even better after all five-community cards have been opened since at this point our player knows for sure if he has a winning hand and can bid accordingly.

So on average our player puts his money in with a better hand and his chances of winning are 75% against his opponents 25%. If his opponent does draw a miracle card or two to save himself, a so-called "bad beat", this is only thanks to chance and so now we have found an element of chance in Poker100. Out of 100% of hands, our player will fold 50% as inferior and win 75% of the remaining 50%, therefore winning 37.5% of total hands. This is less then 50% of hands he would win based on chance alone since by folding inferior hands he is essentially giving up his share of "bad beats" against his opponent. But since the winner in Poker100 is not determined based on the number of hands won, but based on the total amount of units of money won our player is still ahead.

Assuming in half the hands he folds he is a big blind and in all other cases he is a small blind he is losing about 1.5 units of money on all pre-flop folds. He is also losing at least 4 units of money for each "bad beat" hand, which is about 12.5% of total number of hands, generated from betting pre-flop, on the flop and on the turn with the turn bet being 2 units of money as per regular poker rules. No betting takes place on the River since by that time he knows he has a weaker hand. So overall by losing 62.5% of all hands he losses $1.5 * 50 + 4 * 12.5 = 125$ units of money, but by winning 37.5% of all hands by taking them to the showdown he generates $4 * 37.5 = 150$ units of money, for a pure profit of 25 units of money per match, if match consists of 100 hands. If it were not for "bad beats" our player would win 50% of all hands for a cool profit of $4 * 50 = 200$ units or 50 units more which corresponds to $50/200 = 25\%$ element of chance in the game of Poker100, meaning that Poker100 has GSM of 75%.

We have made certain simplifying assumptions in our investigation. First it is very likely that the hands taken to showdown will have much more then 4 units of money in the pot because of all the possible raises and re-raises, but this is compensated by hands which an opponent decided to fold before showdown realizing he is probably beat. We have also assumed that our opponent is not very clever and has not figured out our "optimal" strategy and decided to counteract it. This is a reasonable assumption since we are not concerned with dominance of different strategies over each other all we tried to do was isolate effects of randomness on a game as a whole. Obviously a certain degree of bluffing is necessary to keep our opponent guessing if we really do have a winning hand.

An alternative strategy for measuring the amount of skill in the game of Poker100 is to take randomness completely out of equation. Once in a while in a game of Poker100 both opponents get same exact cards, with possible variance in suites, assuming flush does not play after the flop, they are in exactly the same situation and the player who wins does so because of his superior skill and nothing more. The only way to win the pot with a same hand is to outplay, out bluff, your opponent and eventually make him fold an equal hand. This subset of hands in a game of Poker100, is a game of pure skill and is the lowest bound we can prove on the GSM of Poker100. It is easy to calculate that in Poker100 both players hold equal hole cards about 1.12% percent of the time. Taking into account possible flush draws which make opponents holding not equal (3 or more cards of the same suit on board) this number is further reduces to 1%. So this proves that GSM of Poker100 $\geq$ 1%.

So we have shown by varies means that Poker100 is not a pure game of strategy or a pure game of skill, in fact it is a mixed game. We have established a lower bound of 1% GSM and shown that in reality in long term the GSM of Poker100 is about 75% which means that in Poker100 skill dominates over chance, at least statistically. Some approximate GSM values for many other games are given in the Figure 1.

IV. CONCLUSION

In this paper we attempted to precisely classify games of skill and chance. We investigated current legal definitions for games of skill and chance and proposed a measure targeted at precise description of skill content in any game of mixed type. We have demonstrated and proved application of our skill measure to a very popular variation of poker game, namely Taxes Hold'em. If a government was to issue a precise threshold demonstrating at which point dominance of skill over chance is sufficient to classify a game as a game of skill, we would be able to make a scientific recommendation as to the classification of most games of questionable character. As of right now we have been able to demonstrate statistical predominance of skill over chance in the game of Texas Hold'em in the long run and so would recommend treating it like a game of skill, rather then a game of chance, which it is currently considered to be in most jurisdictions. In general we suggest treating all games with GSM of over 50% as games of skill in the long run.

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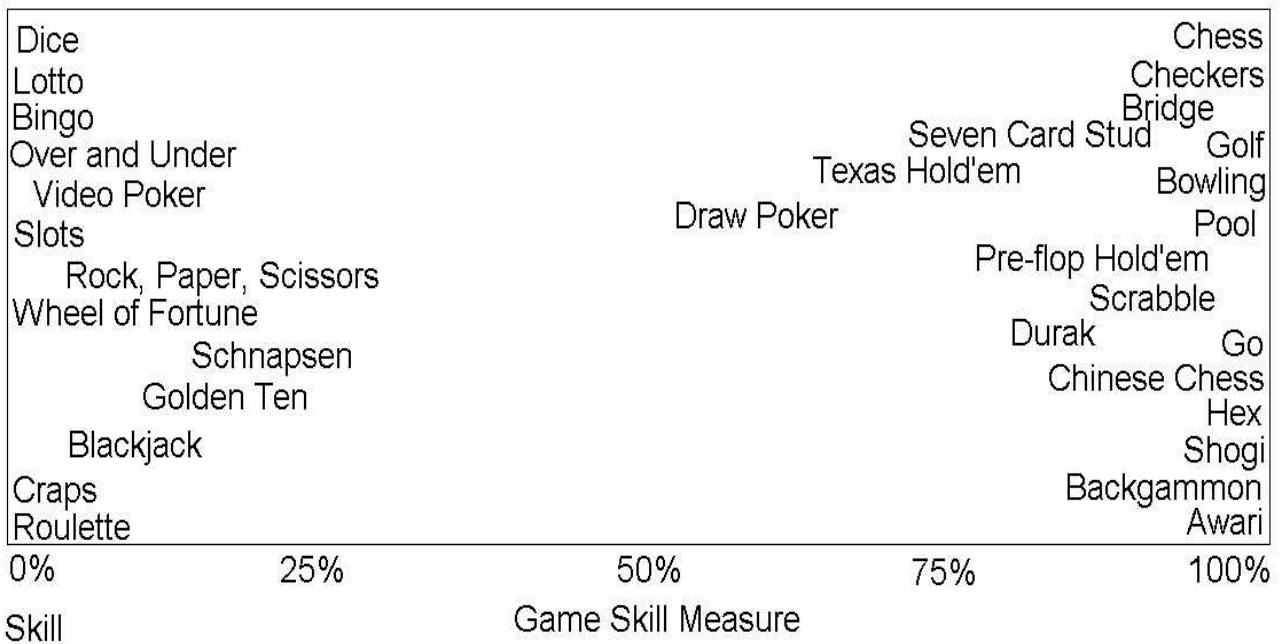
Fig. 1 GSM of some common games