Integrated Maximum Flow Algorithm for Optimal Response Time Retrieval of Replicated Data

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***Abstract*—Efficient retrieval of replicated data from multiple disks is a challenging problem. Traditional retrieval techniques assume that replication is done at a single site using homogeneous disk arrays having no initial load or network delay. Recently, generalized retrieval algorithms are proposed to cover heterogeneous disk arrays, initial loads, and network delays. Generalized retrieval algorithms achieve the optimal response time retrieval schedule by performing multiple runs of a maximum flow algorithm. Since the maximum flow algorithm is used as a black box technique, flow values of the previous runs cannot be conserved to speed up the process. In this paper, we propose integrated maximum flow algorithms for the generalized optimal response time retrieval problem. Our first algorithm uses Ford-Fulkerson method and the second algorithm uses Push-relabel algorithm. Besides the sequential implementations, a multi-threaded version of the push-relabel algorithm is also implemented. Proposed algorithms are investigated using various replication schemes, query types, query loads, disk specifications, and system delays. Experimental results show that the sequential integrated push-relabel algorithm runs up to 2.5X faster than the black box version. Furthermore, parallel integrated push-relabel implementation achieves up to 1.7X speed up (∼1.2X on average) over the sequential algorithm using two threads, which makes the integrated algorithm up to 4.25X (∼3X on average) faster than its black box counterpart.**

***Keywords*-declustering, replication, storage arrays, generalized retrieval, maximum flow, push-relabel**

I. INTRODUCTION

Spatial databases, visualization, and GIS are some of the applications that manage hundreds of terabytes. Many storage products on the market have capacity to store hundreds of terabytes; however, efficient retrieval is still a challenging problem. Traditional retrieval methods based on index structures developed for single disk and single processor environments [28], [30], [37] are ineffective for the storage and retrieval in multiple processor and multiple disk environments. Since the amount of data is large, it is very natural to use parallel disk architectures. Besides scalability with respect to storage, parallel disk architectures offer the opportunity to exploit I/O parallelism during retrieval. The most crucial part of exploiting I/O parallelism is to develop storage techniques that access the data in parallel. *Declustering* is the most common approach for efficient parallel I/O. The data space is partitioned into disjoint regions, and data is allocated to multiple disks. When users issue a query, data falling into disjoint partitions is retrieved in parallel from multiple disks.

Many declustering schemes were proposed assuming a single copy of the data [15], [17], [36], [40], [42], [44]. Recently, replication strategies for spatial range queries [16], [23], [26], [27] and arbitrary queries [35], [39], [41] were proposed. Replication improves the worst-case additive error for declustering using multiple copies of the data. In addition to offering lower worst-case additive error, replication has many other advantages including better fault-tolerance and support for queries of arbitrary shape. Readers are directed to [43] for an in-depth comparison and analysis of replicated declustering schemes.

Optimal response time retrieval problem of replicated data is first formulated in [18] along with a Ford-Fulkerson based maximum flow solution for homogeneous disk arrays located on a single site. [45] extended the problem by locating the homogeneous disk arrays on two different sites and proposed a black box maximum flow algorithm for this extended problem. Generalized optimal response time retrieval problem handling the heterogeneous disk arrays, initial loads of the disks, and more than two number of sites is formulated in [12]. Generalized problem is solved by performing multiple runs of a black box maximum flow algorithm [12]. Although the number of maximum flow calls are optimized using binary capacity scaling technique in [12], black box usage of maximum flow did not allow the conservation of previously calculated flows eliminating possible performance improvements.

Deciding the retrieval schedule of a query is a time critical issue since the decision time is directly added to the response time of the query. In this paper, we propose two sequential, and one parallel integrated maximum flow algorithms to reduce this decision time for the generalized retrieval problem. We found out that integrated push-relabel based algorithms are superior to the integrated Ford-Fulkerson based algorithms for optimal response time retrieval problems. Our sequential integrated push-relabel algorithm runs up to 2.5X faster than the black box version proposed in [12], and the parallel integrated push-relabel implementation achieves up to 4.25X (∼3X on average) speed-up using two threads compared to the black box algorithm.

The rest of the paper is organized as follows. In Section II, we present related background information and motivation behind this work. Section III describes the Ford-Fulkerson based integrated solution. Sequential push-relabel based integrated algorithm is provided in Section IV, and parallel integrated push-relabel implementation is explained in Section V. We evaluate the performance of the proposed algorithms in Section VI and conclude with Section VII.

II. MOTIVATION AND BACKGROUND

In this section, we present the motivation behind this work together with necessary background information.

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A. Application Model

Many applications have data generated at multiple sites and queried by users from multiple sites. Storing all the data at a central site is impractical. Replication of data on multiple storage arrays with distant locations is necessary since multi-server retrieval can be used and the load can be distributed among the servers. An example model is provided in Figure 1, where geographically distant storage arrays are connected over a dedicated network.

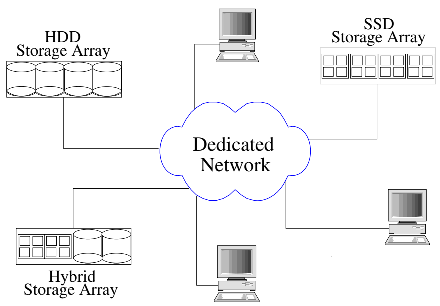


Fig. 1. Storage arrays connected with a dedicated network

Although traditional HDD (Hard Disk Drive) based storage arrays still dominate the market share, SSD (Solid-state Drive) based storage arrays [4], [6], [7], [9] and hybrid storage arrays [3], [5], [8], [10] composed of SSDs and HDDs gain a lot of attention recently. Our model supports the heterogeneity of the storage arrays such that they can be HDD based, SSD based or hybrid. In addition to the heterogeneity of the storage arrays, we also consider the network delay to the storage arrays and the initial load of the disks caused by the previous queries. Many Internet service providers now offer dedicated Internet access with bandwidth, latency, packet loss, and availability guarantees. For example, XO communications' dedicated Internet access [1] guarantees round-trip latency of 65 milliseconds edge-to-edge within the XO network, round-trip packet loss of at most 1%, and availability of 100%. Using the guarantees given by a dedicated network, an estimate network delay to a storage array can be determined. Besides the network delay, initial loads of the disks from the previous queries can also be calculated easily since it is based on how the previous queries are scheduled.

Potential applications of the model are as follows:

- Dataset for an application is stored on a storage array. A new high-end storage array is purchased. Instead of moving all the data to the new storage array, a system spanning the two storage arrays can be used. Storage arrays are costly and making the most out of them is crucial.
- A large dataset is split and stored at storage arrays at multiple sites. We want to run an application that potentially processes parts of the whole dataset. The model above allows us to do this efficiently.
- High-end computer centers with storage arrays can be combined to create storage systems with much larger capacity in an affordable way.
- An SSD based or hybrid storage array is added to a storage system. Since SSDs have write limitations, it is necessary to use them together with other storage arrays and the model above can be used for this purpose.

B. Maximum Flow Problem

Maximum flow is a general technique used in optimal response time retrieval problems [12], [18], [45]. In the maximum flow problem, the goal is to send as much flow as possible between two vertices, subject to edge capacity limits. An instance of the maximum flow problem is a network $G = (V, E, s, t, u)$, where $s \in V$ is a distinguished vertex called the *source*, $t \in V$ is a distinguished vertex called the *sink*, and u is a capacity function. A *flow* is a pseudoflow that satisfies the *flow conservation constraints*:

$$\forall v \in V - \{s,t\} : \sum_{w \in V:(v,w) \in E} f(v,w) = 0 \qquad (1)$$

Equation 1 states that for all vertices except the source and sink, the net flow leaving that vertex is zero. The *value* of a flow f is the net flow into the sink as in Equation 2.

$$|f| = \sum_{v \in V:(v,t) \in E} f(v,t) \qquad (2)$$

The maximum flow problem has been studied for over fifty years. It has a wide range of applications including the transshipment and assignment problems. Known solutions to this problem include Ford-Fulkerson *augmenting path method* [24], [25], the closely related *blocking flow method* [22], [33], *network simplex method* [21], [32], and *push-relabel method* of Goldberg and Tarjan [29].

1) Ford-Fulkerson Method: The motivation behind the Ford-Fulkerson *augmenting path method* is as follows:

An *augmenting path* is a residual *s-t* path. If there exists an augmenting path in G_f, then we can improve f by sending flow along this path. Ford and Fulkerson [24] showed that the converse is also true.

Theorem 1: A flow f is a maximum flow if and only if G_f has no augmenting paths.

This theorem motivates the augmenting path algorithm of Ford and Fulkerson's [24], which repeatedly sends flow along augmenting paths, until no such paths remain.

2) Push-relabel Method: Push-relabel methods send flow along individual edges instead of entire augmenting paths. This leads to better performance both in theory and practice [29]. The push-relabel algorithm works with preflows, which is a flow that satisfies capacity constraints except additional flows into a vertex is allowed called excess. A vertex with positive excess is said to be active. Each vertex is assigned a height, where initially all the heights are zero except $height[s] = |V|$. An iteration of the algorithm consists of selecting an active vertex, and attempting to push its excess to its neighbors with lower heights. If no such edge exists, the vertex's height is increased by 1. The algorithm terminates when there are no more active vertices with label less than $|V|$.

C. Replicated Declustering and Retrieval

A replicated declustering of 7×7 grid using 7 disks is given in Figure 2. The grid on the left represents the first copy and the grid on the right represents the second copy.

Each square denotes a bucket and the number on the square denotes the disk that the bucket is stored at. An $i \times j$ range query has i rows and j columns. For retrieval of an $i \times j$ range query from homogeneous disks, the best we can expect is $\lceil \frac{i*j}{7} \rceil$ disk accesses and this happens if the buckets of the query are spread to the disks in a balanced way. In most cases, this is not possible without replication.

q_1

0	1	2	3	4	5	6
3	4	5	6	0	1	2
6	0	1	2	3	4	5
2	3	4	5	6	0	1
5	6	0	1	2	3	4
1	2	3	4	5	6	0
4	5	6	0	1	2	3

q_1

0	1	2	3	4	5	6
2	3	4	5	6	0	1
4	5	6	0	1	2	3
6	0	1	2	3	4	5
1	2	3	4	5	6	0
3	4	5	6	0	1	2
5	6	0	1	2	3	4

Fig. 2. Orthogonal Allocation

The notation used in this paper along with their meanings is given in Table I.

TABLE I
NOTATION

Notation	Meaning
N	Total number of disks in the system
$\lvert Q \rvert$	Total number of buckets to be retrieved; query size
c	Number of copies for each bucket
C_j	Average retrieval cost of a single bucket from disk j
D_j	Network delay to the server where disk j is located
X_j	Time it takes for disk j to be idle if busy, 0 otherwise

D. Basic Retrieval Problem

In optimal response time retrieval problem, we have N disks and $|Q|$ buckets. Each bucket can be replicated among multiple disks. The aim is to find a way of retrieving the requested buckets of a query from the disks so that the overall response time of the query is minimized. The basic problem assumes that the disks are homogeneous without having any initial load or network delay and they are all located on a single site. In this case, the overall response time of the query is determined by the disk that is used to retrieve the maximum amount of buckets. In other words, we need to retrieve as few buckets as possible from the disk that is used to retrieve the maximum amount of buckets.

Basic problem is solved as a max-flow problem using graph theory. When replication is used, each bucket is stored on multiple disks and we have to choose one of the disks for retrieval of the bucket. Consider the query q_1 given in Figure 2. Query q_1 is a 3×2 query with optimal retrieval cost of $\lceil \frac{3 \times 2}{7} \rceil = 1$. However, since in the first copy the buckets $[0,0]$ and $[2,1]$ are both stored on disk 0, retrieval using the first copy requires 2 disk accesses. When we consider both copies, we can represent the problem as a maximum flow problem [18] as in Figure 3.

For each bucket and for each disk we create a vertex. In addition, two more vertices called source and sink are created. Source vertex s is connected to all the vertices denoting the buckets and all the vertices denoting the disks are connected to the sink vertex t. An edge is created between vertex v_i denoting bucket i and vertex v_j denoting disk j if bucket i is stored on disk j. Next step is to set the capacities of the edges. All the edges except the ones between the disks and the sink have capacity 1. The capacity of the edges between the disks and the sink are set to $\lceil \frac{|Q|}{N} \rceil$.

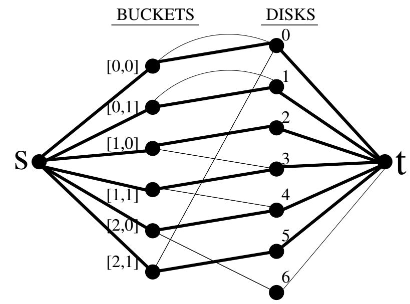


Fig. 3. Max-flow representation of query q_1 for single site

Maximum flow representation of query q_1 for single site is given in Figure 3. Maximum flow is shown using thick lines in the figure. Since query q_1 has only 6 buckets and $\lceil \frac{6}{7} \rceil = 1$, all the edges have capacity 1 in this case. When using maximum flow representation, if the maximum flow between the source and the sink is $|Q|$, then the query can be retrieved using $\lceil \frac{|Q|}{N} \rceil$ disk accesses. Otherwise, we need to increment the capacities of all the edges going to the sink by one and re-run the max-flow algorithm. We repeat this incrementation process until the flow of $|Q|$ is reached. Max-flow algorithm is called $O(|Q|)$ time in the worst case where all the buckets are stored at a single disk.

E. Generalized Retrieval Problem

Besides covering the properties of the basic problem, generalized retrieval problem also handles the heterogeneity of the disks, initial loads, multi-site retrieval, and network delay. Consider the query q_1 given in Figure 2 again but this time assume that the grid on the left represents the allocation at site 1 and the grid on the right represents the allocation at site 2. There are 14 disks in the system, disks 0-6 are located at site 1 and the disks 7-13 are located at site 2. Generalized retrieval problem can be formulated as a maximum flow problem [12]. For the set of parameters summarized in Table II, optimal response time retrieval of the query q_1 is shown in Figure 4 using the thick lines.

Let E be the edge set holding every edge e_j between the disk vertex j and the sink. As in the basic problem, all the edges except the ones in E have capacity 1. In the basic problem, since all the disks are homogeneous without having any network delay or initial load, it is possible to initially set the capacities of all the edges in E to the theoretical lower bound $\lceil \frac{|Q|}{N} \rceil$. If the flow of $|Q|$ cannot be reached, capacities of the edges is E are incremented all at the same time. However, this is not possible for the generalized retrieval problem since different disks might have different retrieval costs depending on their speed, initial load, and network delay. Figure 4 shows the proper values of the capacities for the parameters defined in in Table II.

In order to set the capacities of the edges in E properly, [12] proposes a binary capacity scaling algorithm. The algorithm defines a range where the optimal response time is known to

TABLE II
SYSTEM PARAMETERS

Disk j	C_j (ms)	D_j (ms)	X_j (ms)
0-6	8.3	2	1
7,8,10,13	6.1	1	0
9,11,12	13.2	1	0

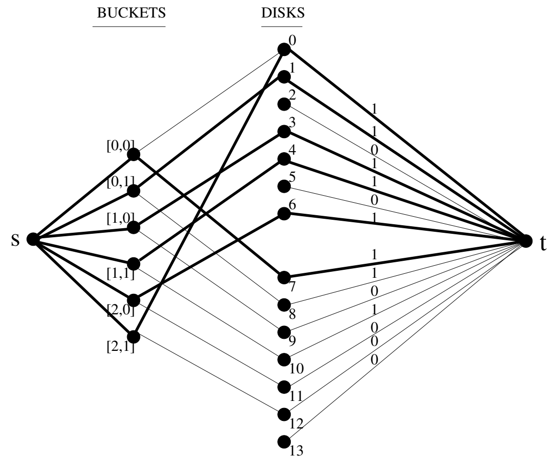


Fig. 4. Max-flow representation of query q_1 for 2 sites

be within this range. In each iteration, the algorithm picks the middle value of this range, calculates the capacities for this middle value and runs the maximum flow algorithm. Depending on the flow value, the algorithm decreases the range by half either eliminating the top range or the bottom range. After the range is small enough, the algorithm reaches to the optimal response time retrieval by incrementing the capacity of the edge yielding the minimum cost and running the max-flow in each iteration. The algorithm terminates when the maximum flow of $|Q|$ is reached. Readers are directed to [12] for more details.

The problem with the solution proposed in [12] is that it uses maximum flow as a black box technique. Therefore, in each run of the maximum flow with different capacities, flow values are calculated starting from zero all over again. Since the algorithm increments the capacities used in one previous run for each step, current run of maximum flow can start from the flow values calculated in the previous run. This approach will obviously save considerable amount of maximum flow calculations; however, it requires an integrated maximum flow solution for the generalized retrieval problem.

III. FORD-FULKERSON BASED SOLUTION

The first integrated algorithm we propose for the generalized retrieval problem uses the Ford-Fulkerson method. Algorithm 1 shows a Ford-Fulkerson based integrated algorithm proposed in [18] for the basic retrieval problem.

Algorithm 1 assumes that flow values of the edges going out of the source vertex are all initialized to 1 at the beginning. Lines 1-2 sets the capacities of the edges in E to the theoretical lower bound $\lceil\frac{|Q|}{N}\rceil$, where E holds all the edges between the disk vertices and the sink. Then, for each bucket i in the query, the algorithm searches for an augmenting path from the vertex representing the bucket i to the sink vertex in lines 3-4. If

Algorithm 1 $FordFulkersonBasic()$

```
1: for all e ∈ E do
2:    caps[e] ← ⌈|Q|/N⌉
3: for i ← 1 to |Q| do
4:    dfs_success = DFS(G, v[i], t, caps, flow, path)
5:    while (!dfs_success) do
6:       for all e ∈ E do
7:          caps[e]++
8:       dfs_success = DFS(G, v[i], t, caps, flow, path)
9:    for all e ∈ path do
10:      if target(e) ≠ t then
11:         G.reverse_edge(e)
12:      if isDirectionReverse(e) then
13:         flow[e]- -
14:      else
15:         flow[e]++
16: fixReversedEdges()
```

Algorithm 2 $FordFulkersonIncremental()$

```
1: for all e ∈ E do
2:    caps[e] ← 0
3: for i ← 1 to |Q| do
4:    dfs_success = DFS(G, v[i], t, caps, flow, path)
5:    while (!dfs_success) do
6:       IncrementMinCost()
7:       dfs_success = DFS(G, v[i], t, caps, flow, path)
8:    for all e ∈ path do
9:       if target(e) ≠ t then
10:         G.reverse_edge(e)
11:      if isReverse(e) then
12:         flow[e]- -
13:      else
14:         flow[e]++
15: fixReversedEdges()
```

Algorithm 3 $IncrementMinCost()$

```
1: min_cost ← MAXDOUBLE
2: for all e ∈ E do
3:    v ← G.source(e)
4:    if G.in_degree(v) ≤ caps[e] then
5:       E.delete(e)
6:    else
7:       cost[e] ← D[e] + X[e] + (caps[e] + 1) * C[e]
8:       if costs[e] < min_cost then
9:          min_cost ← costs[e]
10: for all e ∈ E do
11:    if costs[e] == min_cost then
12:       caps[e]++
```

no augmenting path exists, capacities of the edges in E are incremented by one until a path is found through the lines 5-8. For each edge in the path, line 11 reverses its direction if the edge is between a bucket vertex and a disk vertex. This reversal is necessary to be able to change the retrieval decision of a previously assigned bucket. Finally, lines 12-15 increments or decrements the flow of each edge in the path depending on its direction. If the edge direction is not its original direction, then the flow is decremented meaning that the retrieval choice is changed, otherwise the flow is incremented. At the end of the algorithm, we have to fix the directions of the edges since some of them might have a reverse direction. It is proven in [18] that Algorithm 1 has the worst case complexity of $O(c*|Q|^2)$ since the DFS might try $c*|Q|$ edges in the worst case. Note that the capacity incrementation in lines 6-7 is performed at most $O(|Q|)$ time during the entire execution of the algorithm.

Algorithm 1 works for the basic retrieval problem only. In order to extend this algorithm to the generalized retrieval problem, we propose Algorithm 2. Algorithm 2 starts to the capacity incrementation from 0 (lines 1-2) since is not possible to use a simple formula like $\lceil \frac{|Q|}{N} \rceil$ as a lower bound in the generalized case. Secondly, since each disk might have a different retrieval cost, capacities of the edges in E cannot be incremented at the same time. Only the capacity of the edge yielding the minimum retrieval cost should be incremented in each incrementation step. Algorithm 2 calls Algorithm 3 for this purpose in line 6, which is the main difference between Algorithm 2 and Algorithm 1. Algorithm 3 determines the edges in E yielding the minimum retrieval cost in lines 7-9 and increments the capacities of the edges with this minimum cost in lines 11-12. Note that, if there are more than one edges yielding the same retrieval cost, their capacities are incremented at the same time as in the basic problem. Lines 3-5 remove the edge from the edge set E if the disk associated with that edge cannot be used to retrieve any more buckets. This removal ensures that the number of incrementation steps are bounded by $O(c * |Q|)$ in the worst case. Therefore, worst case complexity of the Algorithm 2 is $O(c^2 * |Q|^2)$.

IV. Push-relabel Based Solution

Although Ford-Fulkerson based algorithms are simple and easy to implement, most of the practical maximum flow implementations are based on push-relabel based algorithms. Therefore, we also propose a push-relabel based integrated maximum flow algorithm for the generalized optimal response time retrieval problem. Algorithm 4 presents a basic push-relabel based maximum flow algorithm. Lines 1-8 show the initialization step. Push/relabel operations of the algorithm are performed in lines 9-10, which we skip for the sake of simplicity; however, readers are directed to [29] for more details. When the algorithm terminates, excess value of the sink holds the maximum flow amount that can be pushed from the source s to the sink t. Our implementation of Algorithm 4 has the complexity of $O(|V^3|)$, where $|V|$ is the number of vertices in G. This is possible since we use the FIFO ordering for selecting vertices and exact height calculation heuristics suggested by [19].

Algorithm 4 $PushRelabelBasic()$

1: **for all out edges(e,s) do**
2:     $v \leftarrow target(e)$
3:     $QUEUE.append(v)$
4:     $flow[e] \leftarrow cap[e]$
5:     $excess[v]$ += $cap[e]$
6: **for all nodes(v,G) do**
7:     $height[v] \leftarrow 0$
8: $height[s] \leftarrow G.number\_of\_nodes()$
9: **while** $QUEUE \neq \emptyset$ **do**
10:     apply push/relabel operations by updating the $QUEUE$
11: **return** $excess[t]$

Algorithm 5 presents a push-relabel based integrated maximum flow algorithm for the generalized response time retrieval problem. Similar to Algorithm 2, Algorithm 5 increments the capacity of the edge yielding the minimum retrieval cost in each incrementation step as in line 2 and runs until the excess value of the sink reaches to $|Q|$. For every iteration, we clear the queue as in line 3 and initialize the height values as in line 11-13. Push-relabel operations ensure that excess values of the vertices except the source and the sink vertex are all 0 when the algorithm terminates. Therefore, we only set the excess value of source to 0 in every iteration as in line 14. Note that our aim is conserving the flows found in the previous runs, therefore flow values are not initialized back to 0. In addition to this, the algorithm should add vertices to the queue only if they can pass more flow in the next push/relabel operations. We check this by using the δ value calculated in line 6 and initialize these vertices only in lines 7-10.

Algorithm 5 $PushRelabelIncremental()$

1: **while** $excess[t] \neq |Q|$ **do**
2:     $IncrementMinCost()$
3:     $QUEUE.clear()$
4:     **for all out edges(e,s) do**
5:         $v \leftarrow target(e)$
6:         $\delta \leftarrow cap[e] - flow[e]$
7:         **if** $d > 0$ **then**
8:             $QUEUE.append(v)$
9:             $flow[e] \leftarrow cap[e]$
10:             $excess[v]$ += $cap[e]$
11:     **for all nodes(v,G) do**
12:         $height[v] \leftarrow 0$
13:     $height[s] \leftarrow G.number\_of\_nodes()$
14:     $excess[s] \leftarrow 0$
15:     **while** $QUEUE \neq \emptyset$ **do**
16:         apply push/relabel operations by updating the $QUEUE$
17: **return** $excess[t]$

Algorithm 5 solves the generalized optimal response time retrieval problem using an integrated push-relabel based maximum flow algorithm. Although we can conserve the flows calculated for the previous capacities in the current run, the algorithm still considers all possible retrieval times starting from the minimum in an exhaustive search manner. Since the graph has $|V| = |Q| + N + 2$ vertices, Algorithm 5 has the worst case complexity of $O(c * |Q|^4)$ where $O(|Q|^3)$ comes from the push/relabel operations (assuming $|Q| > N$) and $O(c * |Q|)$ comes from $IncrementMinCost()$. Since we are conserving the flows, the algorithm is expected to run faster in practice; however, we can still improve the worst case complexity further by using the binary capacity scaling technique presented in [12]. Binary capacity scaling technique will bring the capacity values up to an initial value in $O(log(|Q|))$ operations before the incrementation step is started by Algorithm 5. By this way, number of incrementation steps will be bounded by N, the number of disks in the system.

Algorithm 6 presents our final push-relabel based integrated maximum flow algorithm that uses the binary capacity scaling technique. First, we define a range $[t_{max}, t_{min})$ in lines 1-11 where we know the optimal response time retrieval lies within. In order to ensure this, we calculate t_{max} assuming that all the query buckets are retrieved from the disk with the largest retrieval cost, and t_{min} assuming that all the query buckets are retrieved from the disk with the smallest retrieval cost. Since we want to ensure that there is no solution for t_{min}, we subtract the min_speed value from t_{min} in line 11. min_speed is the average retrieval time of a single block from the fastest disk in the system calculated in lines 9-10.

Algorithm 6 $PushRelabelBinary()$

1: $min\_speed \leftarrow MAXDOUBLE$
2: $t_{min} \leftarrow MAXDOUBLE$
3: $t_{max} \leftarrow 0$
4: **for all** $e \in E$ **do**
5:     **if** $D[e] + X[e] + |Q| * C[e] > t_{max}$ **then**
6:         $t_{max} \leftarrow D[e] + X[e] + |Q| * C[e]$
7:     **if** $D[e] + X[e] + \lceil \frac{|Q|}{N} \rceil * C[e] < t_{min}$ **then**
8:         $t_{min} \leftarrow D[e] + X[e] + \lceil \frac{|Q|}{N} \rceil * C[e]$
9:     **if** $C[e] < min\_speed$ **then**
10:         $min\_speed \leftarrow C[e]$
11: t_{min} -= min_speed
12: **while** $(t_{max} - t_{min}) \geq min\_speed$ **do**
13:     $t_{mid} \leftarrow t_{min} + (t_{max} - t_{min}) * 0.5$
14:     **for all** $e \in E$ **do**
15:         $caps[e] \leftarrow \lfloor (t_{mid} - D[e] - X[e])/C[e] \rfloor$
16:     $QUEUE.clear()$
17:     **for all out edges(e,s) do**
18:         $v \leftarrow target(e)$
19:         $\delta \leftarrow cap[e] - flow[e]$
20:         **if** $d > 0$ **then**
21:             $QUEUE.append(v)$
22:             $flow[e] \leftarrow cap[e]$
23:             $excess[v]$ += $cap[e]$
24:     **for all nodes(v,G) do**
25:         $height[v] \leftarrow 0$
26:     $height[s] \leftarrow G.number\_of\_nodes()$
27:     $excess[s] \leftarrow 0$
28:     **while** $QUEUE \neq \emptyset$ **do**
29:         apply push/relabel operations by updating the $QUEUE$
30:     **if** $excess[t] \neq |Q$ **then**
31:         $StoreFlows()$
32:         $tmp\_excess\_t \leftarrow excess[t]$
33:         $t_{min} \leftarrow t_{mid}$
34:     **else**
35:         $RestoreFlows()$
36:         $excess[t] \leftarrow tmp\_excess\_t$
37:         $t_{max} \leftarrow t_{mid}$
38: $RestoreFlows()$
39: $excess[t] \leftarrow tmp\_excess\_t$
40: **for all** $e \in E$ **do**
41:     $caps[e] \leftarrow \lfloor (t_{min} - D[e] - X[e])/C[e] \rfloor$
42: $PushRelabelIncremental()$

Algorithm 6 finds the capacities of the flow graph for t_{mid} in line 15, performs initialization of the push/relabel operations through the lines 16-27, and applies push/relabel operations in lines 28-29. If there is no solution such that $excess[t] \neq |Q|$, we store the current flow state of the graph as in lines 31-32 to be used later to eliminate unnecessary flow calculations. Also, we increase t_{min} to t_{mid} as in line 33 to reduce the range. If there is a solution such that $excess[t] = |Q|$, since we cannot be sure of the optimality of the solution, we restore the saved flow values and decrease t_{max} to t_{mid} as in lines 35-37. The algorithm stops when the range is smaller than min_speed, restores the saved flows, calculates the final capacities using t_{min}, and calls $PushRelabelIncremental()$ through the lines 38-42. Worst case complexity of the Algorithm 6 is $O(log(|Q|) * |Q|^3)$; however, it is expected to perform better in practice thanks to the flow conservation property.

V. Parallel Push-relabel Implementation

Most new generation storage arrays are powered with multi-core processors. Since retrieval decision is a time critical issue, it is reasonable to use multi-threaded implementations in order to reduce the execution time of retrieval algorithms further. Many push-relabel based parallel maximum flow algorithms are proposed in the literature such as [13] and [14]. However, both of these implementations require locks to perform push/relabel operations. Since locks are known to have expensive overheads [20], we decided to use the asynchronous parallelization method presented in [31]. The algorithm presented in [31] uses the same push/relabel techniques proposed in [29]; however, it does not require any locks or barriers to protect the push/relabel operations. Instead, they use atomic read-modify-write instructions. We implemented a parallel version of our Algorithm 6 using pthreads and the techniques described in [31]. Since the parallelization should take place in the push/relabel operations, line 29 of the Algorithm 6 is modified to support multi-threaded push/relabel operations as it is described in [31].

VI. Experimental Results

In this section, we provide experimental results using various sets of parameters. Our aim is comparing the execution times (running time, runtime) of the proposed algorithms with the existing ones. We investigate the impact of allocation scheme, query type, query load, disk speed, network delay to the site, and initial load of the disks on the execution times. We implemented the experiments in C++ and compiled using g++ version 4.4.3 optimization level 1 (-O1) using the graph structure of LEDA [34] except the parallel implementation. For the parallel generalized retrieval algorithm, we used C and the pthread library in order to comply with the original implementation presented in [31]. Parallel implementation is compiled using gcc version 4.4.3 optimization level 3 (-O3) as in [31]. The machine we used has dual Intel Xeon X5672 quad-core processors with total of 8 cores. Each core has 3.2 GHz of clock speed and the machine has 32GB of physical memory running on an Ubuntu 10.04.03 LTS operating system.

A. Allocation Scheme

We have experimental results for three different allocation schemes described below.

- *Random Duplicate Allocation:* Random Duplicate Allocation (RDA) [38] stores a bucket on two disks chosen randomly from the set of disks. Retrieval cost of random allocation is at most 1 more than the optimal with high probability for single site retrieval [38].
- *Orthogonal Allocation:* Orthogonal allocations [23], [39] guarantee that when the disks that a bucket is stored at is considered as a pair, each pair appears only once in the disk allocation. In an $N \times N$ declustering system with N disks, there are N^2 buckets and N^2 pairs. So, it is possible to have each pair exactly once. For the first copy, we used the threshold based declustering scheme [44].

- *Dependent Periodic Allocation:* A d-dimensional disk allocation scheme $f(i_1, i_2, \ldots, i_d)$ is *periodic* if $f(i_1, i_2, \ldots, i_d) = (a_1 * i_1 + a_2 * i_2 + \ldots + a_d * i_d) \bmod N$, where N is the number of disks and each a_i $i = 1 \ldots d$ satisfies $gcd(a_i, N) = 1$ and $a_i \neq 0$ [11], [46]. For the first copy, we use the allocation scheme yielding the lowest additive error based on the results provided in [11]. For the second copy, a shifted version of the first copy is used. The two allocations are in the form of $f(i,j) = a_1 * i + a_2 * j \mod N$ and $g(i,j) = f(i,j) + m \mod N,\ 1 \leq m \leq N-1$.

B. Query Types

We have experimental results for two different types of queries; range queries and arbitrary queries.

- *Range Query:* Range queries are rectangular in shape. We assume a wraparound grid consistent with the choice of disk allocations. A range query is identified with 4 parameters (i, j, r, c) $0 \leq i, j \leq N-1, 1 \leq r, c \leq N$. i and j are indices of the top left corner of the query and r, c denote the number of rows and columns in the query. The number of distinct range queries on an $N \times N$ grid is $(\frac{N*(N+1)}{2})^2$, which can be found by counting the number of ways to choose two points out of $N+1$ row and column points on the grid as follows: $\binom{N+1}{2} * \binom{N+1}{2}$.
- *Arbitrary Query:* Arbitrary queries have no geometric shape. Any subset of the set of buckets is an arbitrary query. We can denote arbitrary queries as a set and the number of arbitrary queries is $\sum_{i=1}^{N^2} \binom{N^2}{i}$ which is equal to 2^{N^2} (number of subsets of a set with N^2 elements).

C. Query Load

We use three different query loads.We use the notation p_k^i to denote the probability that a query in load i can be retrieved in k disk accesses optimally. Once the optimal number of disk accesses k is selected, the number of buckets is selected uniformly from the range $[(k-1)N+1, kN]$.

- *Load 1:* The distribution of queries is similar to the distribution of queries for the particular query type. For the distribution of range queries, smaller size queries are more likely; for the distribution of arbitrary queries, medium size queries are more likely. Expected bucket size of load 1 queries is $\frac{N^2}{4} + O(\frac{1}{N})$ for range queries and $\frac{N^2}{2} + O(\frac{1}{N})$ for arbitrary queries.
- *Load 2:* The distribution of queries is uniform. We achieve this by setting $p_k^2 = \frac{1}{N}$. Expected bucket size of load 2 queries is $\frac{N^2}{2}$.
- *Load 3:* Much smaller queries than load 1 and load 2 are more likely. We achieve this by setting $p_k^3 = \frac{2^N}{(2^N-1)*2^k}$. In this case $p_k^3 = \frac{1}{2}p_{k-1}^3, 2 \leq k \leq N$. Expected bucket size of load 3 queries is $\frac{3N}{2}$.

D. Disks

We have experimental results on five different disks. Specifications of the disks are given in Table III. The time value provided in the table is the average access time to read a block in our system, which is calculated using the Ubuntu disk utility benchmark. Average access time of a block is defined as the summation of *spin-up time*, *seek time*, *rotational latency* and *transfer time* for HDDs; just *transfer time* for SSDs.

TABLE III
DISK SPECIFICATIONS

Producer	Model	Type	RPM	Time (ms)
Seagate	Barracuda	HDD	7.2K	13.2
WD	Raptor	HDD	10K	8.3
Seagate	Cheetah	HDD	15K	6.1
OCZ	Vertex	SSD	-	0.5
Intel	X25-E	SSD	-	0.2

E. Experiment Parameters

All the experiments conducted are summarized in Table IV. Delay and initial load values are given in milliseconds. R(2,10,2) means that a number among the set 2, 4, 6, 8, and 10 is chosen randomly. If the system is homogeneous, the properties of the cheetah disk is used for all the disks in the system. If the system is heterogeneous, then the disks are chosen randomly among the disk group indicated in the table. Disk groups can be HDDs, SSDs, or HDDs+SSDs.

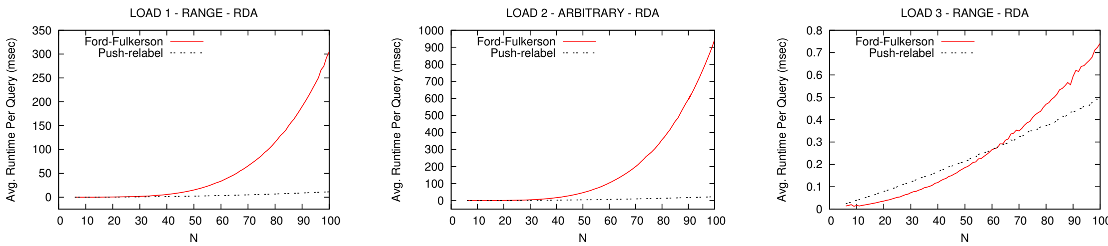
TABLE IV
EXPERIMENTS

Exp. Num.	# of Sites	Disk Prop.	Site 1			Site 2		
			Disks	Delays	Loads	Disks	Delays	Loads
1	2	hom.	cheetah	0	0	cheetah	0	0
2	2	het.	ssd	0	0	hdd	0	0
3	2	het.	hdd	0	0	ssd	0	0
4	2	het.	ssd+hdd	0	0	ssd+hdd	0	0
5	2	het.	ssd+hdd	R(2,10,2)	R(2,10,2)	ssd+hdd	R(2,10,2)	R(2,10,2)

F. Results

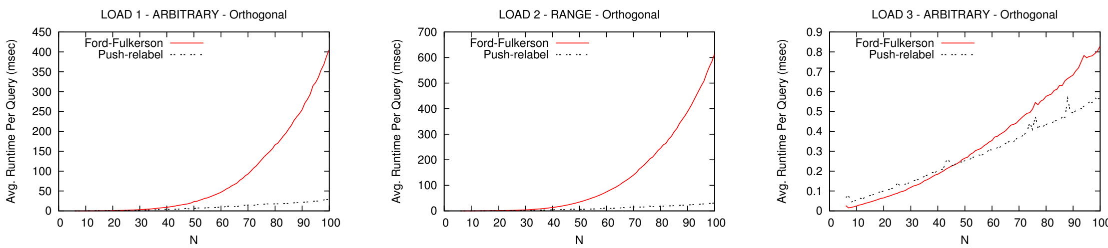
In this section, we provide some of the experimental results that are interesting for our purposes. All the results are available on the project web page [2]. In all experiments, we used an $N \times N$ grid for N disks and for each value of N, 1000 queries are performed. For every experiment we performed, we compared the total optimal response time values of these 1000 queries for each algorithms we tested and found out that the results are matching as expected. In this paper, we are interested in the execution time comparison of the algorithms. Therefore, we do not share the response time values of the experiments. An in depth study for the effect of different parameters on the response time of the queries can be found in [12].

1) Ford-Fulkerson vs. Push-relabel: In this section, we compare the execution times of the Ford-Fulkerson based algorithms with the Push-relabel based algorithms. Since the basic retrieval problem is a subset of the generalized retrieval problem, algorithms proposed for the generalized retrieval problem can also solve the basic retrieval problem. Experiment 1 is an example of a basic retrieval problem where the disks are homogeneous and there is no initial load or network delay. Figure 5 compares the Algorithm 1 with the Algorithm 6 using the Experiment 1, where x-axis shows the number of disks and y-axis shows execution times. Even though Ford-Fulkerson based retrieval algorithms have better worst case complexity,



(a) Range, Load 1 (b) Arbitrary, Load 2 (c) Range, Load 3

Fig. 5. Experiment 1, RDA, Ford-Fulkerson - Push-relabel Execution Time



(a) Arbitrary, Load 1 (b) Range, Load 2 (c) Arbitrary, Load 3

Fig. 6. Experiment 5, Orthogonal, Ford-Fulkerson - Push-relabel Execution Time

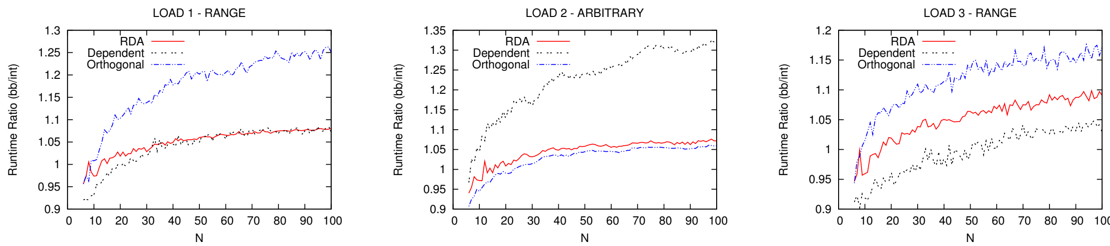
push-relabel based retrieval algorithms are better in practice. Execution time of the push-relabel based Algorithm 6 is at most 25 milliseconds for load 2 with $N = 100$ and $|Q| = 5000$; however, Ford-Fulkerson based Algorithm 1 requires almost a second for same case. The results are similar for load 1. Since the query sizes are small for load 3, Algorithm 1 is slightly better than Algorithm 6 for $N < 60$ (less than 0.1 milliseconds difference); however, Algorithm 1 does not scale well when the number of disks and the query size increase.

Figure 6 compares the Ford-Fulkerson based Algorithm 2 with the Push-relabel based Algorithm 6 for the generalized retrieval problem of Experiment 5. Performances are similar to the basic retrieval case shown in Figure 5. For Experiment 5, push-relabel based Algorithm 6 requires at most 30 ms when $N = 100$ and $|Q| = 5000$. Figure 5 and Figure 6 clearly show that push-relabel based retrieval algorithms are superior to the Ford-Fulkerson based retrieval algorithms for both the basic retrieval problem and the generalized retrieval problem.

2) Blackbox Push-relabel vs. Integrated Push-relabel: In this section, we compare the execution time of the black box push-relabel based binary retrieval algorithm proposed in [12] with our integrated binary push-relabel algorithm presented in Algorithm 6. Black box algorithm uses LEDA's push-relabel implementation and the integrated algorithm modifies the same implementation for fair results. Figure 7 shows this comparison for the basic retrieval case using Experiment 1, where x-axis shows the number of disks and the y-axis shows the execution time ratios (black box/integrated). Since capacities of the disks are incremented at the same time in the basic retrieval case, less capacity incrementation steps are performed. Therefore, flow conservation property of the integrated algorithm could not be exploited. Nonetheless, if a certain allocation scheme requires more capacity incrementation, then the integrated algorithm clearly outperforms the black box algorithm. For example, Orthogonal allocation requires more incrementation steps for the range queries and RDA requires more incrementation steps for the arbitrary queries. In these cases, integrated algorithm performs about 1.3X faster than the black box algorithm. By this way, performance gap between different allocation schemes are also decreased thanks to the integrated algorithm. This can be observed more clearly in Figure 8.

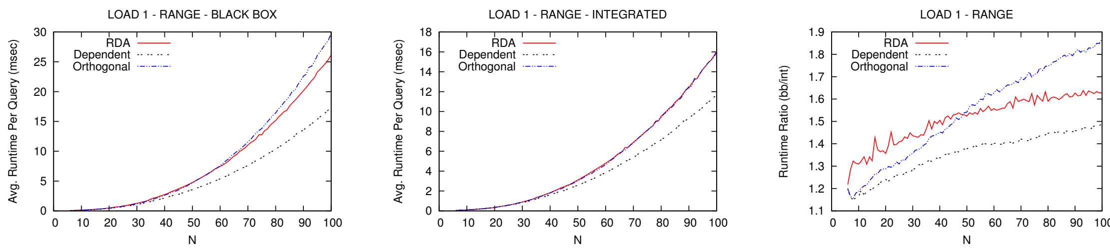
Figure 8 compares the black box and the integrated push-relabel based retrieval algorithms using the Experiment 3. Figure 8(a) shows the execution time of the black box algorithm. Retrieval algorithm for dependent allocations have lesser execution time since their retrieval choices are more obvious than Orthogonal allocations and RDA. However, execution times for Orthogonal and RDA are expected to be similar. Figure 8(b) shows the execution time of the integrated algorithm. As it is clear from the graph, execution time for Orthogonal and RDA are similar and execution time for dependent is slightly less than the others as expected. In other words, integrated algorithm decreased the execution time gap between different allocation schemes. Therefore, execution time ratio of black box and integrated algorithm presented in Figure 8(c) is higher for the Orthogonal allocation.

Figure 9 shows the execution time comparison of the black box and the integrated algorithm for the Experiment 5, where we see the most dramatic performance improvement of the integrated algorithm. Integrated algorithm is up to 2.5 times faster than the black box algorithm. As the number disks and the query sizes increase, performance improvement of the integrated algorithm also increases. The reason for this performance improvement lies in the flow conservation property of the integrated algorithm. Retrieval decision is harder in Experiment 5 since all the parameters are random. Therefore, Experiment 5 requires more incrementation steps



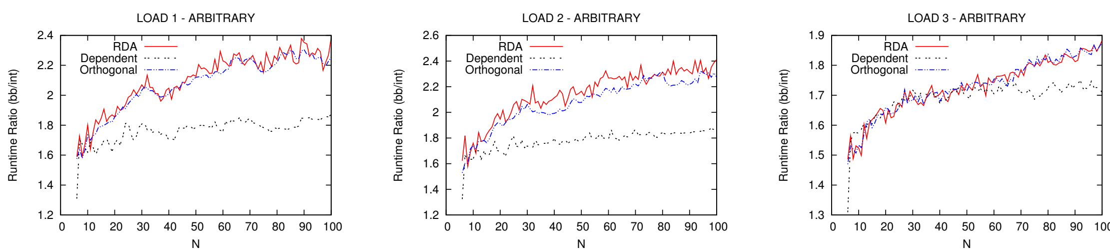
(a) Range, Load 1 (b) Arbitrary, Load 2 (c) Range, Load 3

Fig. 7. Experiment 1, Push-relabel, Black Box/Integrated Execution Time Ratio



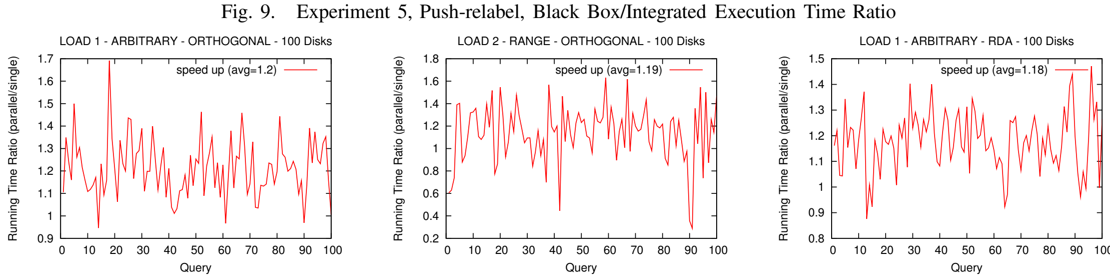
(a) Black Box Execution Time (b) Integrated Execution Time (c) Execution Time Ratio

Fig. 8. Experiment 3, Arbitrary Load 1, Push-relabel Algorithms Comparison



(a) Load 1 (b) Load 2 (c) Load 3

Fig. 9. Experiment 5, Push-relabel, Black Box/Integrated Execution Time Ratio



(a) Arbitrary, Load 1, Orthogonal (b) Range, Load 2, Orthogonal (c) Arbitrary, Load 1, RDA

Fig. 10. Experiment 5, Push-relabel Parallel/Sequential Execution Time Ratio, 2 threads, 100 disks

than the other experiments. Since there is no flow conservation in the black box algorithm, each maximum flow calculation starts with zero flows. On the other hand, integrated algorithm conserves the flow values calculated in the previous run and uses them in the current run.

3) Sequential Push-relabel vs. Parallel Push-relabel: Figure 10 shows the execution time comparison of the push-relabel based retrieval algorithm presented in Algorithm 6 with the parallel implementation of the same algorithm using Experiment 5. Since the performance of the parallel maximum flow algorithm is highly dependent on the graph structure [31], we show different queries on the x-axis and the execution time ratio (parallel/sequential) in the y-axis. Parallel algorithm is executed using two threads for 100 disks and gains a maximum speed-up of 1.7X (∼1.2X on average) over the sequential implementation. The fluctuation in the graph is caused by the change in the graph structure depending on the query size. For small queries of load 3 and more than two number of threads, we observed a load-balancing issue among the threads. Together with this improvement, integrated algorithm becomes 4.25X (∼3X on average) faster than the black box implementation proposed in [12].

VII. Conclusion

In this paper, we propose integrated maximum flow algorithms for the generalized retrieval problem where heterogeneous disks with initial loads can be located on multiple sites having different network delays. Our first algorithm is based on the Ford-Fulkerson method and the second algorithm is based on the Push-relabel technique. We investigate the execution time of the proposed and existing algorithms for the basic and the generalized retrieval problems. Experimental results using various replication schemes, query types, query loads, disk specifications, site delays, and initial disk loads show that proposed Push-relabel based algorithms are superior to the Ford-Fulkerson based algorithms. In addition to this, integrated push-relabel based algorithm is up to 2.5X faster than the existing black box counterpart. We also implemented a parallel version of the proposed push-relabel based integrated algorithm and observed a speed-up of 1.7X maximum (~1.2X on average) over the sequential algorithm. Together with the parallel implementation, our proposed integrated algorithm runs up to 4.25X (~3X on average) faster than the existing black box algorithm.

VIII. Acknowledgments

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